CPA Maths Calculation Policy 2022-2023

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy. It is working document and will be revised where deemed necessary. Many variations of CPA have been included for each strand to equip teachers with a range of tools to support the pupils in their understanding of number and calculation.

To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy and the Joseph Turner Maths Vocabulary Progression document.

What is CPA in maths?

CPA stands for the 'Concrete Pictorial Abstract' approach which is a system of learning that uses physical and visual aids to build the understanding of abstract topics.

Pupils should be introduced to new mathematical concepts through the use of concrete resources (e.g. cubes, base10/Dienes, multilink cubes, counters). When they are secure in solving problems using the visual aids, they move on to looking at problems in a pictorial representation (usually pictorial representations of concrete objects they were using).

The concrete and pictorial concepts **will not** provide the answers for the children. It will help support, scaffold and make clear mathematical concepts.

Children are then asked to solve problems where they only have an abstract representation i.e. numbers or other symbols. Building these small steps across a lesson, can help pupils better their understanding of the relationship between numbers and the real world. In turn, this helps secure their understanding of the mathematical concept they are learning.

'Concrete' Representation

New concepts are introduce through the use of physical objects or practical equipment. These resources can be physically handled, enabling children to explore different mathematical concepts. These are sometimes referred to as manipulatives and can be specific mathematical resources (e.g. dienes, counters) or ordinary household items (e.g. straws, dice, fruit).

The abstract nature of maths can be confusing for children, but through the use of concrete materials, children are able to 'see' and make sense of what is actually happening.

Misconception

'Concrete resources are only for lower attainers'

'Concreate resources only need to be used in lower key stages'

Actually, concrete resources can be useful for children at every stage in their learning journey. All children, regardless of age or ability, benefit from the use of practical resources in ensuring understanding goes further than learning a procedure.

Practical resources promote reasoning and discussion which enable all learners to articulate and think deeper about a concept. Teachers will be able to observe children gaining a greater understanding of where misconceptions lie and establish the depth of their understanding.

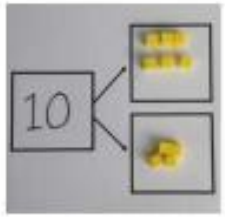
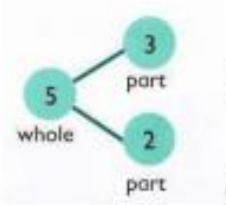
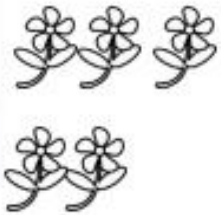
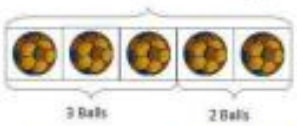
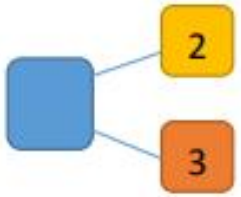
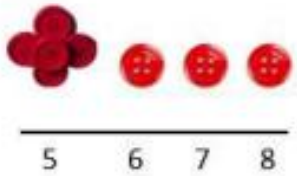
'Pictorial' Representation

Once children are confident with a concept of using concrete resources, they progress to drawing pictorial representations or quick sketches of the objects. They will be no longer manipulating the physical resources but will still have the benefit of the visual support the resources provide.

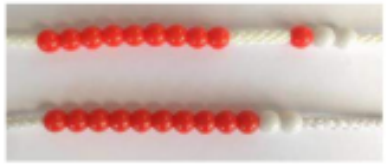
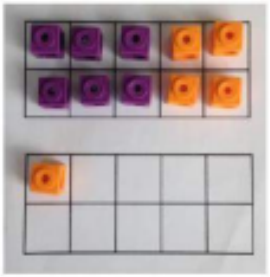
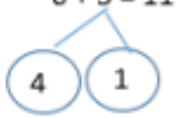
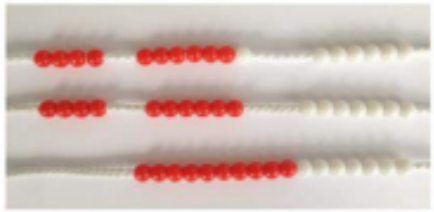
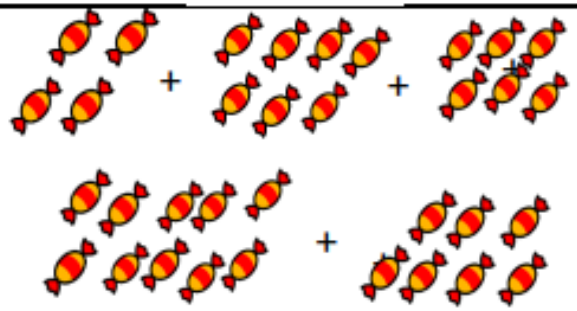
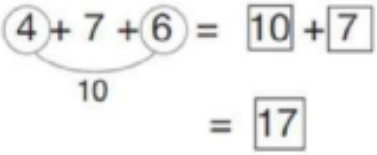
Abstract

Once children have a secure understanding of the concept through the use of concrete resources and visual images, they are then able to move on to the abstract stage. Children will be using abstract symbols to model problems – using numerals. To be able to access this stage effectively they need to have had experience of the concreate and pictorial.

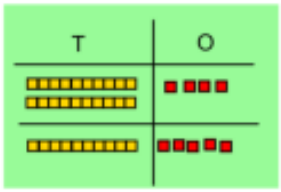
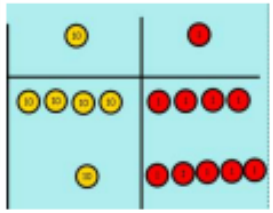
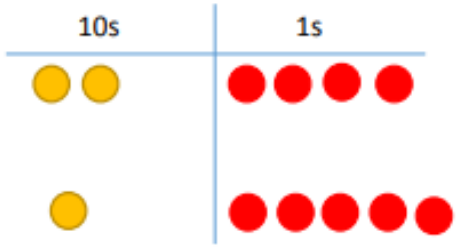
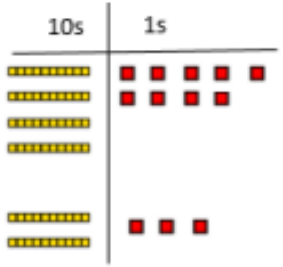
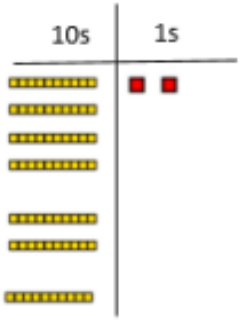
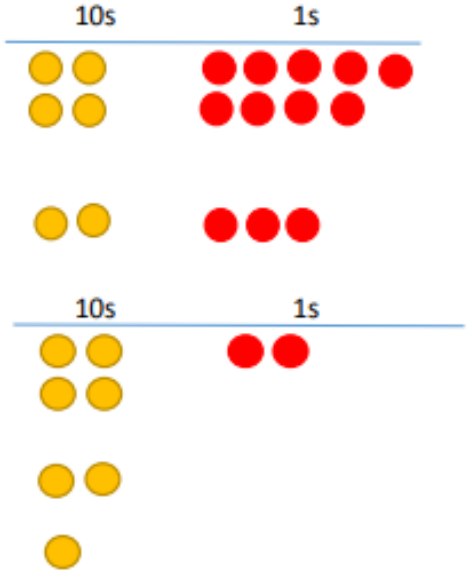
Calculation Guidance - Addition

	Objective	Concrete	Pictorial	Abstract
Year 1	Number bonds of 5, 6, 7, 8, 9 and 10	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3  Use the part-part-whole diagram as shown above to move into the abstract.
	Counting	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	Use a number line to count on in ones. 	5 + 3 = 8

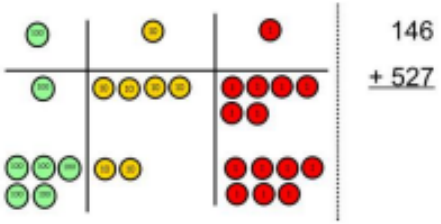
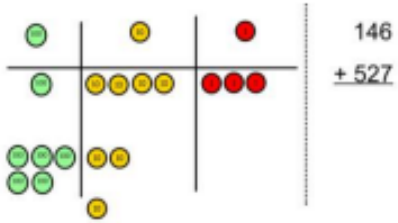
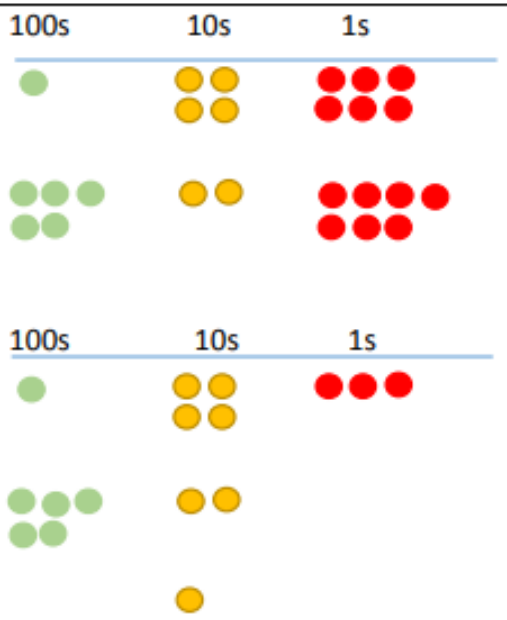
Calculation Guidance - Addition

	Objective	Concrete	Pictorial	Abstract
Year 1	Regrouping to make 10	  6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	 6 + 5 = 11  6 + 4 =10 10 + 1 = 11	6 + 5 = 11
Year 2	Adding 3 single digit numbers	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

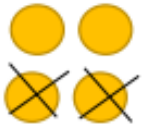
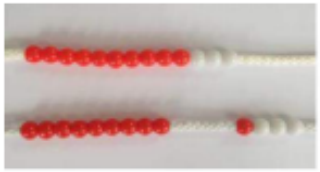
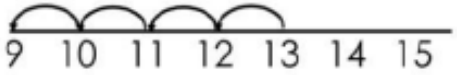
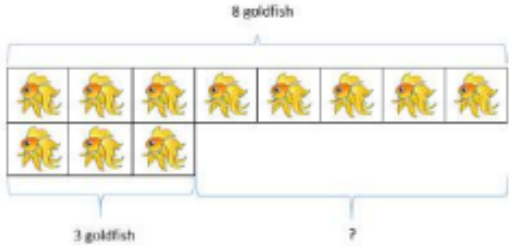
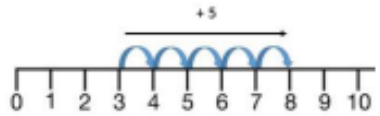
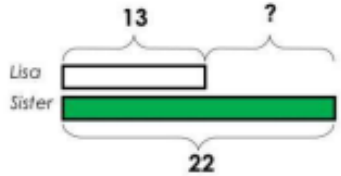
Calculation Guidance - Addition

	Objective	Concrete	Pictorial	Abstract
Year 2	Column method without regrouping	Add together the ones first, then add the tens. Use the Base 10 blocks first before moving onto place value counters. 24 + 15 =  44 + 15 = 	After physically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	24 + 15 = 39 24 + 15 39
	Column method with regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten.  	Using place value counters, children can draw the counters to help them to solve additions.  	40 + 9 20 + 3 60 + 12 = 72

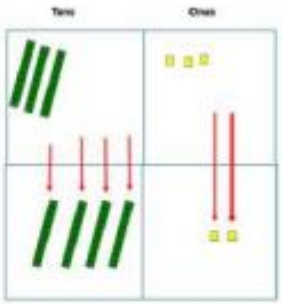
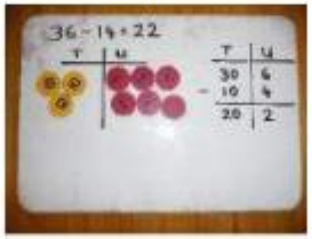
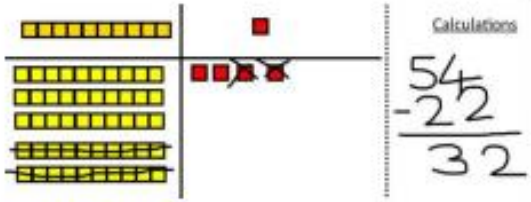
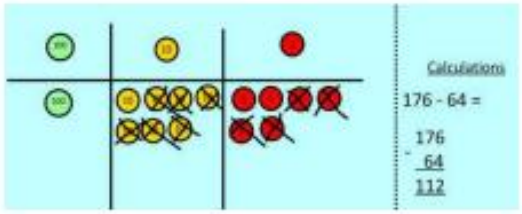
Calculation Guidance - Addition

	Objective	Concrete	Pictorial	Abstract
Year 3/4	Column method with regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten.  As children move on to decimals, money and decimal place value counters can be used to support learning. **NB** By Year 4 children will progress on to adding four digit numbers.	 Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. **NB** Addition of money needs to have £ and p added separately.	100 + 40 + 6 500 + 20 + 7 600 + 70 + 3 = 673 As the children progress, they will move from the expanded to the compacted method. 146 + 527 673 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.
Year 5/6	Column method with regrouping	Consolidate understanding using numbers with more than 4 digits and extend by adding numbers with up to 3 decimal places.		

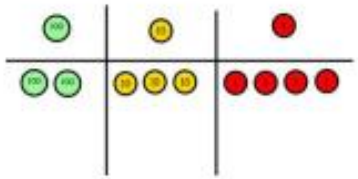
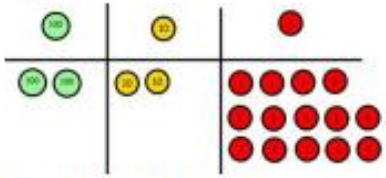
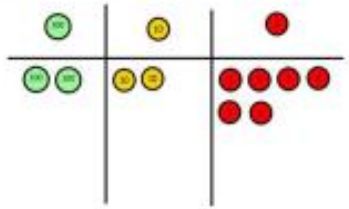
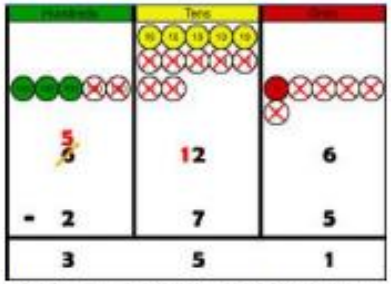
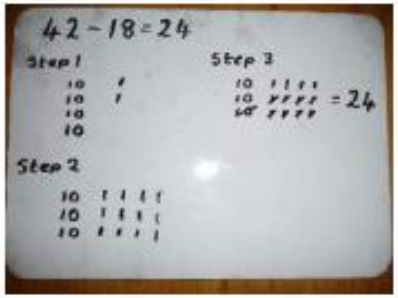
Calculation Guidance - Subtraction

	Objective	Concrete	Pictorial	Abstract
Year 1	Taking away ones	Use physical objects, counters, cubes etc. to show how objects can be taken away. 4 – 2 = 2 	Cross out drawn objects to show what has been taken away. 4 – 2 = 2 	4 – 2 = 2
	Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 = 9	Count back on a number line or number track  Start at the bigger number and count back the smaller number, showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
	Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference.	 Count on to find the difference. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Draw bars to find the difference between 2 numbers.	Hannah has 8 goldfish. Helen has 3 goldfish. Find the difference between the number of goldfish the girls have.

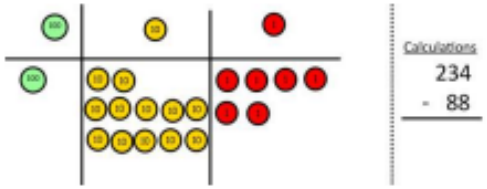
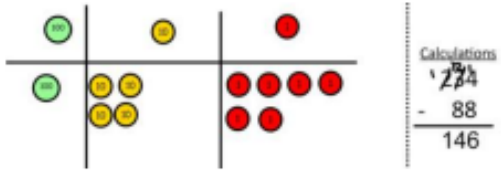
Calculation Guidance - Subtraction

	Objective	Concrete	Pictorial	Abstract
Year 2	Column method without regrouping	75 – 42 = 33  Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 This will lead to a clear written column subtraction. 

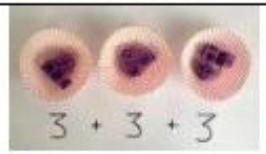
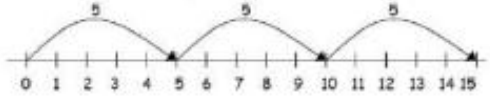
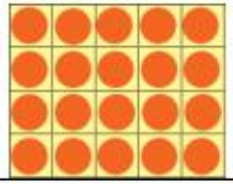
Calculation Guidance - Subtraction

	Objective	Concrete	Pictorial	Abstract
Year 3 onwards	Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.  Calculations 234 - 88 Now I can subtract my ones.  Calculations 234 - 88	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup. 	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5

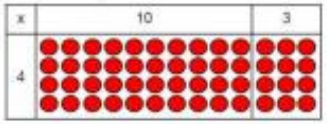
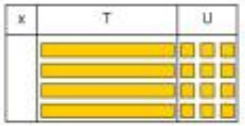
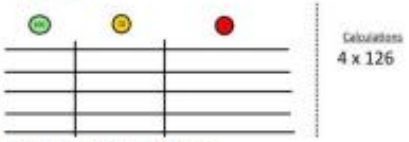
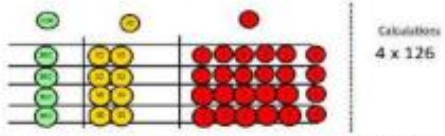
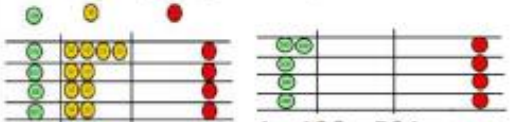
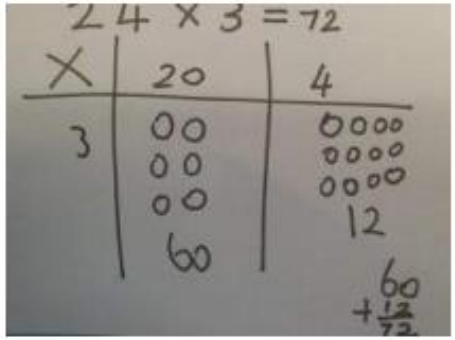
Calculation Guidance - Subtraction

	Objective	Concrete	Pictorial	Abstract
Year 3 up	Column method with regrouping	Now look at the tens, can I take away 8 tens easily? I need to exchange 1 hundred for 10 tens.  Calculations 234 - 88 Now I can take away 8 tens and complete my subtraction.  Calculations ~~2~~ 1 ~~3~~ 4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Calculation Guidance – Multiplication

	Objective	Concrete	Pictorial	Abstract
Year 1/2	Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 + 2 + 2 = 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 = 6
	Arrays- showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8 Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Calculation Guidance – Multiplication

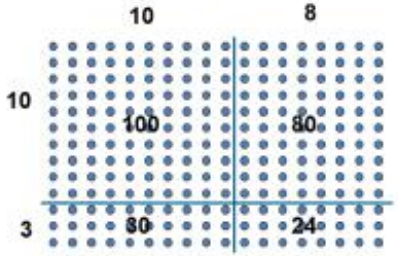
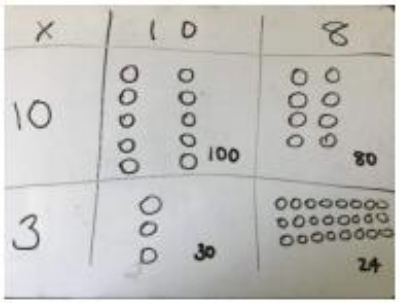
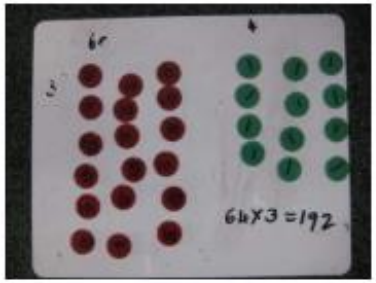
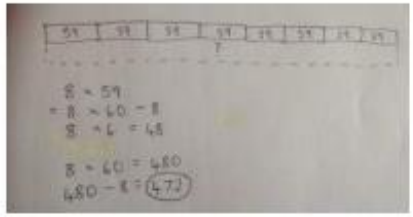
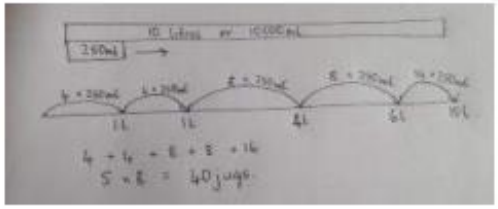
	Objective	Concrete	Pictorial	Abstract
Year 3/4	Grid method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  4 × 126 = 504	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

×	30	5
7	210	35

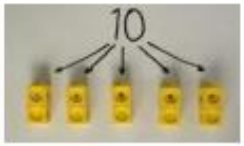
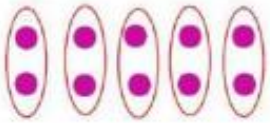
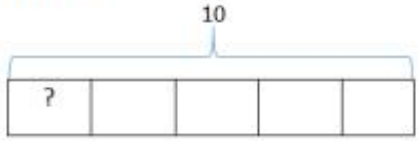
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

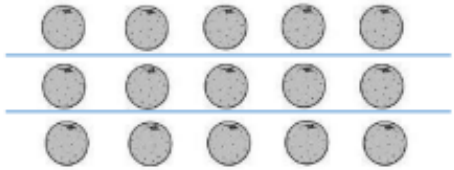
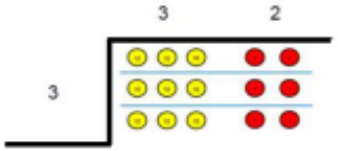
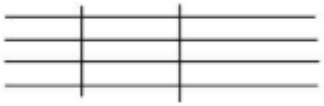
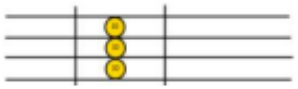
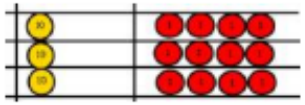
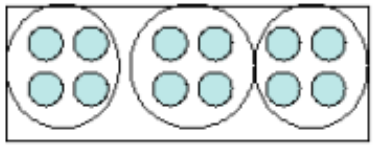
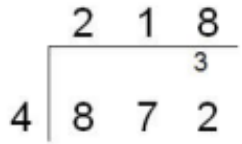
Calculation Guidance – Multiplication

	Objective	Concrete	Pictorial	Abstract
	Expanded method	Show the link with arrays to first introduce the expanded method. 		Start with long multiplication, reminding the children about lining up their numbers clearly in columns. 18 x 13 24 (3 x 8) 30 (3 x 10)) 80 (10 x 8) 100 (10 x 10) 234
Year 5/6	Compact method	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 74 x 63 = 4662 12 210 240 + 4200 4662 This moves to the more compact method. 1342 x 18 13420 10736 24156

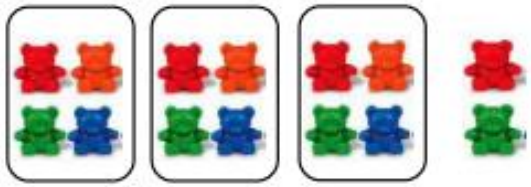
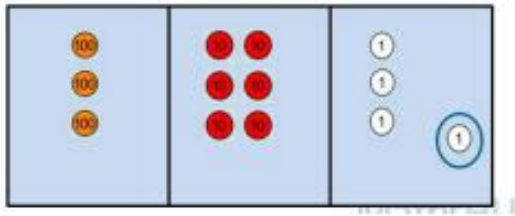
Calculation Guidance – Division

	Objective	Concrete	Pictorial	Abstract
Year 1/2	Sharing	I have 8 cubes, can you share them equally between two people?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 8 buns between two people. 8 ÷ 2 = 4 
	Grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  10 ÷ 5 = ? 5 x ? = 10	10 ÷ 5 = 2 Divide 10 into 5 groups. How many are in each group?

Calculation Guidance – Division

	Objective	Concrete	Pictorial	Abstract
Year 3/4	Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5
	Short division	Use place value counters to divide using the short division method alongside. 96 ÷ 3  42 ÷ 3 Start with the biggest place value.   We are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for 10 ones and then share the ones equally among the groups.  We look at how many are in each group. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  2 1 8 3 4 8 7 2

Calculation Guidance – Division

	Objective	Concrete	Pictorial	Abstract
Year 5/6	Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 
	Short division with remainders	364 ÷ 3 = 1 2 1 rem 1 3 | 3 6 4 		Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or decimal according to the context. 8 6 r 2 3 5 | 4 3 2 1 8 6 1/5 5 | 9 4 3 1 1 4 . 6 16 21 3 5 | 5 1 1 . 0

Calculation Guidance – Division

	Objective	Concrete	Pictorial	Abstract
Year 6	Long division			Children will use long division to divide numbers with up to 4 digits by 2 digit numbers. 015 32 ⟌ 487 -0 48 -32 167 -160 7 17 r 19 31 ⟌ 546 31↓ 236 217 19